The Years of Peak Astronomical Tide

This investigation stems from an obscure statement by the late D. H. Macmillan in a semi-popular book on tides. He stated that the years when the tide-raising forces reach their greatest peak values occur at intervals of very roughly 16 centuries, and gave a sequence of dates starting with 3500BC so and ending with 1433 and 3300AD. He did not describe the exact terms of the calculation but merely suggested that they involved coincidence of perihelion (closest approach of the Earth to the Sun) and zero solar declination (Sun crossing equator), which is in fact impossible within the given range of dates. (The longitude of perihelion crosses the equator at intervals of about 105 centuries and will next do so in 6581AD). Further, if one computes the lunar and solar elements for 3300AD, one finds that they are not particularly favourable for large tides.

To clarify the situation I must first resolve a paradox. By simple consideration, the greatest tide-raising forces must occur when the Moon and the Sun are in conjunction with the Earth and are at their closest respective distances. But it is also common knowledge that the greatest ‘perigee spring tides’ (spring tides coinciding with a close approach of the Moon) occur at the equinoxes when the Sun crosses the equator), which as mentioned above cannot coincide with the closest approach to the Sun around the present millennium. The solution lies in the fact that the equinoctial condition maximises only the semi-diurnal components of the tide at the expense of the diurnal components. Since most parts of the ocean have a magnified response to the semidiurnal frequencies, one must therefore look to the equinoxes rather than perihelion for the greatest ocean tides. Then all the major harmonic constituents μ_2, $2N_2$, N_2, M_2, L_2. S_2, K_2 are nearly in phase, and the only loss due to the slightly more than minimum distance of the Sun would be an unfavourable phase for the constituent T_2, which is practically negligible because it has a typical amplitude of 2 or 3 cm of water (about 6% of the mean solar tide). On the other hand, if we are more interested in the maxima of the tide-raising forces themselves, or in movements of the Earth's crust which responds to them more directly than the ocean, then position relative to the equator becomes irrelevant and we should confine attention to the times of perihelion. The dates when both types of maxims occurred are considered separately below.

I restricted the computations to the years 1 to 4000AD. Outside this range, the accepted formulae become unreliable and interest in the problem diminishes. For the times of peak tide raising forces, the computer was asked to select the years for which the following conditions all occurred simultaneously: (a) Earth at perihelion (closest to Sun); (b) longitude of Moon's perigee within a small angle ε of perihelion or of its converse (aphelion); and (c) longitude of Moon's ascending node within ε of perihelion or of its converse. The Moon's actual (mean) longitude was also computed at the given times, to check whether the angular limits were not violated by the time needed to adjust to the nearest appropriate conjunction with the Sun (the conjunction very near to perigee). Standard formulae, as found in the Astronomical Ephemeris, were used for the four relevant mean longitudes, With $\varepsilon=5^{\circ}$, perihelion satisfied the above conditions in only the following years: 1340, 1433, 1526, 1619, 3182, 3275, 3368 and 3461.

The appearance of the year 1433 shows that this was the type of calculation behind Macmillan's dates, but 3300 does not appear and there is no suggestion of a 1,600 year cycle. On the contrary, the salient features are sequences of exactly 93 years occurring in groups of four in alternate millennia (also confirmed by tentative calculations for the first millennium BC). The 93 year period arises because it is close to 5 times the 18.61 year period of revolution of the Moon's nodes and 10.5 times the 8.85 year period of perigee - condition (b) permits half periods. But, as the basic periods are strictly incommensurate, one cannot expect any exact cycle of repetition.

The configurations on most of the eight occasions above are, however, somewhat spoilt by the longitude of the Moon itself. If the Moon's 'anomaly' (its angular distance from perigee) is more than about 60º at the given instant, an adjustment of 5 to 15º is required to define the optimum position, during which time the Sun moves by as many degrees as days. The only cases for which the Moon's anomaly and its angular distance from the Sun are less than 60º are in the years 1340 and 3182. Only in these two years does the peak tide-raising force truly occur within 5º of perihelion, perigee and a node.

Turning now to the case of peak semi-diurnal tides, the relevant computations follow a similar pattern, except that in the descriptions of conditions (a), (b) and (c), 'perihelion' is replaced by 'equinox'. (That is, I take the Sun over the equator instead of at its least distance from Earth.) Since equinoxes occur twice as often as perihelion, the conditions tend to be satisfied more frequently. The interval of 4.4 years between passages of the longitude of perigee past the equinoxes, (condition (b) has been observed in the study of extreme low (or high) predicted sea levels. It might be thought that condition (c) should be confined to the autumnal equinox, so that the 18.61 year 'nodal modulation' gives the constituent M_2 its greatest amplitude, but in fact when the Moon passes the equator, M_2 is in phase with K_2, which has a correspondingly reduced amplitude, and other 'nodal modulations' are similarly self-cancelling. So both equinoxes are equally valid in condition c.

With $\varepsilon = 5º$, the three conditions are satisfied on 27 occasions in 1 to 4000 AD. Only those for which the Moon is within 60º of the appropriate conjunction are shown in the table below, for reasons stated earlier. V represents the vernal and A the autumnal equinox.

Most likely years of peak tides			
135A	1020V	2192A	3002V
	1113V	2732V	
	1745A	2825V	
	1922A		

This time no cyclic pattern emerges, either in Table 1 or in the complete list of 27 dates. Intervals of 93 years occasionally appear (for example, 1020-1113 and 2732-2825) for the same reason as before, but the use of the fixed reference of the equinox instead of the slowly moving perihelion renders the 93 year cycle less persistent.

The conditions for 1922 were particularly favourable, since the Moon was less than 1º off 'change' (the instant when 'New Moon' commences) at the time of the equinox and the other relevant angles were also very small. Accurate computations of the lunar and solar ephemerides show that the peak semi-diurnal tide-raising force occurred 1922 September 21 0539 GMT, with amplitude 1.8728 times the mean lunar amplitude (M_2) or 1.2782 times the mean 'spring' amplitude (M_2+S_2). Actual ocean tides usually reach their peak a little later than the peak tidal forces, because of the peculiarities of their dynamic response. Among tide-gauge records from 1922 which are ready to hand, Brest reached a peak of 1.72 × M_2 range, (1.28 × mean 'spring'range) on the tide of September 22 p.m. to September 23 a.m., while Newlyn reached 1.64 × M_2 range (1.22 × mean ‘spring’ range) on the same tide.

Dynamic response may well, of course, cause the tide at a given place to reach its ultimate peak in other years. Places with large diurnal components would again require a quite different set of conditions.

This subject has recently taken on a surprising topicality because of widespread press reports (some grossly exaggerated) of forecasts of unusually large tides for certain days in 1974. The situation is that on January 8 the Moon's perigee occurred within less than 2 hours of Full Moon, while the Earth was within 5º of perihelion. This is close to the conditions for peak tide-raising forces but the fact that the Moon's line of nodes was some 16º away from the Sun disqualifies if from my more rigorous conditions. Being some 72º off the spring equinox, the. (semi-diurnal) oceanic tides are not extraordinarily large and a glance through the tide tables of recent years confirms that equally high and low waters were forecast, for example, for 1966 and 1972. The Full Moons of February and March 1974 occur successively later than perigee, but the approach of the equinox of March 21 compensates to make the spring tides about equally high in all three months. A similar sequence occurs in July, August and September when perigee occurs near New Moon but this time the Sun is not unusually close. The year 1974 is perhaps remarkable for its number of fairly large tides but if one were to compute the occurrences of equal or greater tides in 4,000 years, one would obtain a very long list.

D. E, CARTWRIGHT
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